

1 per cent when (8) or (9) is used. It is recommended that the ratio of height to width of the enclosure be in the order of one, or greater, to ensure this accuracy for any reasonable choice of the other parameters. Errors in measurement are not included in this figure. Where any question arises regarding the error involved the bounds, (6) and (7) may be used. In the high frequency range the accuracy obtained using (8) or (9) can be expected to fall off in some installations, such as the one described. If the standing waves can be avoided on the line an estimate of the error can be obtained using

(19). In the very low frequency range the quality of the enclosure as a shield determines the validity of (8) and (9). By making a relative measurement of the field at two points, and by using the average as an estimate of the error in this range.

ACKNOWLEDGMENT

The author wishes to thank Messrs. E. I. Hawthorne, F. T. Nicholson, and B. Epstein, for their invaluable suggestions and assistance. This work was done under Contract NObsr 49128 with the Bureau of Ships.



Correspondence

Comment on "Instantaneous Frequency"*

R. W. Johnson's note regarding instantaneous frequency in the correspondence columns of the PROCEEDINGS OF THE I.R.E.¹ is noted.

With the plethora of comments about instantaneous frequency which had been appearing in these columns, and noting a discussion of the subject in the *Proceedings of Commission VI of URSI*, at the Xth General Assembly in Sydney, Australia, in August, 1952, I wrote to Dr. B. van der Pol, President of that Commission, in December, 1953, calling his attention to the discussion in these columns. His following reply of February, 1954 leaves no question about his position in the matter.

"Thank you for your letter of December 15th. It was very kind of you to draw my attention to the letters published in the correspondence columns of the PROCEEDINGS OF THE I.R.E.; they had escaped my notice.

"I have now gone through them and noted that they concern the idea of 'Instantaneous Frequency,' which I introduced in radio some 25 years ago in *PROC. I.R.E.*, vol. 18, p. 1194; 1930, of which I am sending you a reprint under separate cover.

"I am fully confident that my definition was right, proper and useful, and still is."

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Comment on "Properties of Some Wide-Band Phase-Splitting Network"*

D. G. C. Luck¹

J. S. LeGrand of our Laboratories recently made use of some RC 90-degree-phase-difference networks of the type shown in Fig. 2(c) and Fig. 2(e) of Dr. Luck's very useful article. When the design equations given in Figs. 2(c) and 2(e) were used, unsatisfactory circuit performance resulted.

To double-check the design equations, I have independently analyzed the two circuits involved and find that the improper performance was a result of the following errors in Dr. Luck's design equations.

In Fig. 2(c) the equation for both attenuation and required magnitude of the 180°-phase-shifted generator should be:

$$k = \frac{1}{2} \frac{(1 - 2Q)}{(1 + 2Q)} \quad (1)$$

In Fig. 2(e) the number two (2) is missing from six of the equations and I will rewrite the corrected equations below:

$$Q = \frac{1}{2} \frac{1}{(R_3/R_2 + 1)} \quad (2)$$

$$k = \frac{1}{2} \left(\frac{1 - 2Q}{1 + 2Q} \right) \quad (3)$$

$$R_2 = 2R_3/[1/Q - 2] \quad (4)$$

$$R_1 = 2R_3/[1/Q^2 - 4] \quad (5)$$

$$C_2 = \frac{1}{2}[1/Q - 2]C_3 \quad (6)$$

$$C_1 = \frac{1}{2}[1/Q^2 - 4]C_3 \quad (7)$$

Incidentally, it may be helpful to other readers to point out the fact that the design parameters f_1 , f_2 and Q of Dr. Luck's useful article can be obtained as follows:

1. Obtain the Smithsonian Elliptic Function Tables by C. W. and R. M. Spenceley.

2. From your required $f_{\max}$ and $f_{\min}$ calculate

$$(a) \quad \theta = \sin^{-1} \sqrt{1 - (f_{\min}/f_{\max})^2}$$

3. If θ is not an exact integral number of degrees modify $(f_{\min}/f_{\max})$ slightly so as to make θ be an integer and still satisfy your requirement. (This procedure removes need for double interpolation in above tables.)

4. Using the above tables calculate

$$(b) \quad a_1 = \frac{cn\theta(K\theta/8)}{sn\theta(K\theta/8)}$$

$$(c) \quad a_3 = \frac{cn\theta(5K\theta/8)}{sn\theta(5K\theta/8)}$$

5. Finally obtain f_1 , f_2 and Q from

$$(d) \quad \frac{f_2}{f_{\max}} = \frac{f_{\min}}{f_1} = \sqrt{a_1 a_3}$$

$$(e) \quad \frac{1}{Q} = \left(\sqrt{\frac{a_1}{a_3}} + \sqrt{\frac{a_3}{a_1}} \right)$$

The background material for the above procedure is contained in the basic articles by Darlington² and Orchard.³

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² S. Darlington, "Realization of a constant phase difference," *Bell Sys. Tech. Jour.*, vol. 29, pp. 94-104; January, 1950.

³ H. J. Orchard, "Synthesis of wideband two-phase network," *Wireless Eng.*, pp. 72-81; March, 1950.

* Received by the IRE, August 4, 1954.

¹ Vol. 42, p. 1024; June, 1954.

* Received by the IRE, July 21, 1954.

¹ *Proc. I.R.E.*, vol. 37, pp. 147-151; Feb. 1949.

Correspondence

A Note on the Fourth Dimension

It may be time that the definition of time be re-examined. The arbitrary intervals set up as standards may have acted as a bottleneck in progress to fully describe the physical environment.

Consider that the passage of time is a function of the changing state of the matter being described. This hypothesis broadens the accepted concept to include nonuniform time changes in general (and in particular, only an approximation of perfect uniformity). The entropy of an ensemble is a measure of its exact organization state. Any change in this entropy is therefore a passage of time.

This entropy is a function of the system internal probabilities so that when they remain unchanged (within the accuracy of measure), then no time has passed for the system. This implies that time is not the fourth dimension and that relative entropy is. (Relative entropy is used to define the static time of the system considered in relation to its final state and thereby it will indicate "age" toward "death" or "uselessness.") The comparison of "times" of different objects may be readily covered by considering each in relation to the entire containing system.

It is considered possible to have negative change of "time" or a decrease in entropy for a particular subject when energy with information is being supplied and the subject is taken to be the entire system in question. The inevitable increase in universal entropy is reflected in the fact that "time stands still for nothing." Yet as far as humans can measure, things of constant probability and entropy are timeless, such as a coin flip or a rock.

A consideration of dimensional analysis should be met early in the proposed concept of relative entropy as the fourth dimension. Since both entropy and relative entropy are both dimensionless quantities and the first three dimensions are measured in arbitrary units, the dissimilarity must be overcome. Two methods are available. Either correct relative entropy to a proper length unit of measure by multiplying by some absolute constant (such as was accomplished in multiplying time by the velocity of light) or reduce the first three dimensions to unitless quantities. Since no absolute length seems defined by nature, the first possibility appears impractical (as well as arbitrary). The second, however, seems made to order. If the system bounds are chosen before attempting to write number values of description, then relative terms may be used throughout. The first three dimensions would be a measure of the ratio of the distance from the origin to the position described, to the distance to the boundary of the system in each direction. The fourth dimension would similarly be a

measure of the ratio of the entropy being considered to the maximum entropy of the system.

The quantities thus far considered are the static dimensional values. Since that which is usually of concern is the change in dimensions, the dynamic values must be examined. Just as $\Delta l = l_2 - l_1$ and $\Delta t = t_2 - t_1$ so it should be defined that $\Delta h = h_2 - h_1$ where h represents the relative entropy $H/H_{\max}$. Next, one unnecessary restriction previously incumbent seems to have been overcome: the instantaneous rate of time change was uniform; however, the instantaneous rate of relative entropy change (the relative entropy gradient) is nonuniform and therefore more explicit.

The special case of periodic variation of relative entropy may be of interest. Consider an amplitude varying information containing signal. Fourier Analysis will break it down into sinusoidal variations in relative entropy (in the entropy domain).

A rather unusual shaped curve would appear (Fig. 1) upon plotting a single sinusoidal component at some constant "frequency" (angular rate of vector rotation ω of x_r). This is due to the nonuniformity of h . The sinusoidal waveshape could be preserved if ω were such a function of h that the product ωh would be linear.

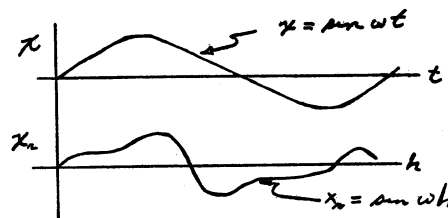


Fig. 1

It might also be profitable to find the relation between h and t . This would describe the real nonuniform time in terms of the fictitious uniform old definition of time.

Since physical entropy is a direct measure of information, the specification of any point in accordance with the proposed system falls into values of relative position in space and relative information. This abides

with the complete universal specification of any system in terms of material, energy and information (with no time measure). It should also be pointed out that since R (the redundancy) $= 1 - (H/H_{\max}) = 1 - h$, then the redundancy is also a measure of the true "time" of the system. It is also important that the quantized states of energy enforces the concept of discontinuous "time" change together with all its implications. It is suggested that natural biological activity might be more readily specified as variations in entropy rather than cycles in arbitrarily uniform time which is astronomical and completely unrelated to the living organism.

After this concept had been set down it was found that some related ideas were discussed in the literature. In particular A. S. Eddington, in his "The Nature of the Physical World," stated the following:

"Having convinced ourselves that the two things are connected, (time and entropy-gradient) we must conclude that there is something as yet ungrasped behind the notion of entropy—some mystic interpretation, if you like—which is not apparent in the definition by which we introduced it into physics. In short we strive to see that entropy-gradient may really be the moving on of time (instead of vice versa). (p. 95)

"Entropy-gradient is then the direct equivalent of the time of consciousness (a mental time concept) in both its aspects. Duration measured by physical clocks (time-like interval) is only remotely connected. (p. 101.)

"Temporal relation and entropy-gradient, both rigorously defined in physics, are entirely distinct and in general are not numerically related." (p. 102)

It is felt that there remains considerable difference between the implications of these statements and the concept proposed in this paper.

This note is presented to indicate a line of reasoning which may or may not prove fruitful. It should be considered in the light of relativity and information theory as well as the basic laws of classical mechanics before final evaluation may be performed.

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TABLE I
Parameters and Symbols

	Standard	Proposed
Relative position to origin	x, y, z	$\frac{x}{x_{\max}} = x_r, y_r, z_r$
Change of position	$\Delta x, \Delta y, \Delta z$	$x_{r2} - x_{r1} = \Delta x_r, \Delta y_r, \Delta z_r$
Time (absolute)		H of the system
Relative time	t	$\frac{H}{H_{\max}} = h = g(t).$
Gradient time	$\frac{dx}{dt} = \text{const.}$	$\frac{dx_r}{dh} = \text{variable}$
Frequency	$f = \frac{\sin^{-1} x}{2\pi t}$	$f = \frac{\sin^{-1} x_r}{2\pi h}$

Correspondence

High Latitude Ionospheric Observations Using Extra-Terrestrial Radio Waves*

The scintillation of localized extra-terrestrial radio sources^{1,2,3} and the absorption of the diffuse background of extra-terrestrial radio waves⁴ have been used by various workers to investigate the ionosphere at temperate latitudes. This letter describes some preliminary observations of the effect of the ionosphere upon the incoming radio waves at College, Alaska, situated approximately 275 kms. south of the auroral maximal zone as defined by Vestine.

The apparatus used in this investigation consisted of a stable, high-gain receiver and a directional antenna, the output of the receiver being recorded continuously by a pen recording instrument. The antenna, a twin Yagi array with beam widths between half power points of approximately 26 degrees and 50 degrees in the two principal planes, was directed at azimuth 215 degrees east of true North and elevation 70 degrees, the beam being oriented so that it was narrow in azimuth and broad in elevation.

Owing to the rotation of the earth, the beam swept out each day a broad strip of the sky which included the localized sources in Cassiopeia and Cygnus (declinations +58 degrees 32' and +40 degrees 35' respectively). The stability of the receiver gain was monitored by switching the receiver from the antenna into a dummy antenna every half hour for a period of 20 seconds—in this way, any drifts due to changes in the gain of the receivers (drifts which might otherwise be attributed to changes in the received antenna power) were revealed by corresponding drifts in the positions of these periodic reference levels.

The records obtained in this way over a two-month period commencing March 16, 1954, have been inspected for two main effects: (a) absorption of the diffuse background of extra-terrestrial radio waves and (b) scintillation of the intense localized radio sources in Cassiopeia and Cygnus.

ABSORPTION

In the absence of ionospheric effects, the intensity of the extra-terrestrial radio waves received by a fixed antenna at a given frequency is a function only of sidereal time. The degree of absorption of the diffuse background of extra-terrestrial radio waves may therefore be determined from the ratio of the signal actually received to the signal received at that sidereal time under condi-

tions of negligible ionospheric absorption. The curve showing the sidereal variation of extra-terrestrial radio power for this equipment is shown in Fig. 1 and was obtained by averaging seven records taken on days when ionospheric sounding equipment, and equipments monitoring distant radio stations, showed that ionospheric absorption was unusually low. The scatter between the records on quiet days was small (rms deviation of signal power at a given sidereal time less than 2 per cent); it is considered that any residual absorption of 65 mc signal on these days was less than 1 per cent.

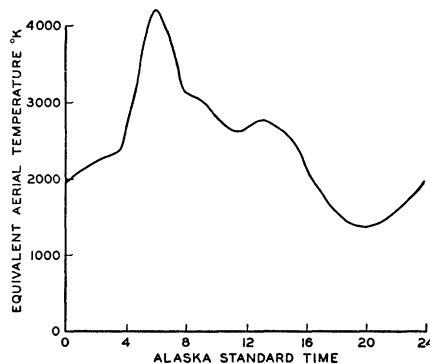


Fig. 1—Diurnal variation of received extra-terrestrial radio power. (The time marks are those for May 10, 1954).

The degree of absorption was measured every half hour throughout the two-month period by comparing each day's record in turn with the standard no-absorption record obtained as above.

The frequency of occurrence of strong absorption at 65 mc (absorption in excess of 10 per cent) is plotted in Fig. 2 as a function of the time of day. It will be seen that such absorption is most common at midday, and occurs least frequently during the late evening. This 65 mc absorption was found to be strongly correlated with no-echo conditions (polar blackouts) on the C-3 Ionospheric Sounding Equipment, the two phenomena showing a similar diurnal variation (Fig. 2), and similar day-to-day variations. (Fig. 3).

In Fig. 4, the daily sum of the three-hour Geomagnetic K indexes for College, Alaska, are compared with the number of half-hourly 65 mc observations per day showing absorption greater than 10 per cent. The correlation coefficient between the two phenomena was 0.48, indicating a strong link between ionospheric absorption and the variability of the earth's magnetic field.

SCINTILLATION

The scintillation work described here, although limited to two months' observations at high angles of elevation, is sufficient to indicate that marked differences exist between the scintillation phenomena in England and Alaska. Thus, scintillations are more common in Alaska than in England and apparently do not show the same diurnal variation of activity. (In England,

zenithal fluctuations are found to be a night time phenomenon and are rarely observed after 0800 G.M.T.; in Alaska, the fluctuations occurred with equal frequency during the four hours preceding, as in the four hours immediately following, 0800 Alaska Standard Time.) Another difference was the variability of the high latitude fluctuations whose characteristics usually changed considerably during a three-hour transit period on several occasions the transit of a source was undisturbed except for one (or occasionally two) short bursts of fluctuations.

One point of considerable interest is that the scintillations were always observed whenever polar blackouts or marked 65 mc absorption occurred during the transits of the localized radio sources through the aerie beam. Since "radio star" scintillations are caused by the presence of localized irregularities in the electron density of the ionospheric layers, this must mean that the lateral distribution of ionization during periods

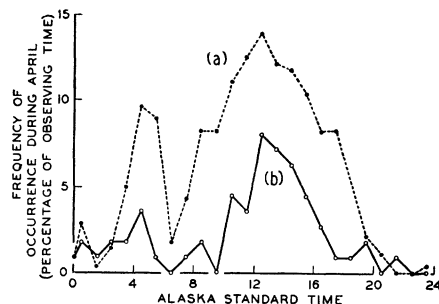


Fig. 2—Diurnal variation of (a) occurrence of polar blackouts and (b) of strong (i.e., greater than 10 per cent) absorption of extra-terrestrial radio waves.

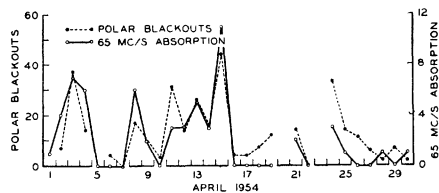


Fig. 3—Daily frequency of detection of polar blackouts (120 observations per day) and strong 65 mc absorption (48 observations per day) during April, 1954.

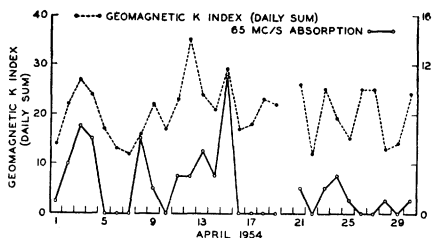


Fig. 4—Comparison of daily sum of three-hour Geomagnetic K indexes for College, Alaska, and number of half-hourly 65 mc observations per day showing absorption greater than 10 per cent.

of absorption is nonuniform. It should therefore be possible, by applying existing techniques to the study of these scintillations, to obtain information about the size, shape, stability, and movements of the

* Received by the IRE, June 18, 1954. This work has in part been financed by the National Science Foundation.

¹ C. G. Little and A. Maxwell, "Fluctuations in the intensity of radio waves from galactic sources," *Phil. Mag.*, vol. 42, pp. 267-278; March, 1951.

² A. Hewish, "The diffraction of galactic radio waves as a method of investigating the irregular structure of the ionosphere," *Proc. Roy. Soc. A*, vol. 214, pp. 494-514; October 9, 1952.

³ A. Maxwell and C. G. Little, "A radio-astronomical investigation of winds in the upper atmosphere," *Nature*, vol. 169, pp. 746-747; May 3, 1952.

⁴ C. A. Shain, "Galactic radiation at 18.3 mc/s," *Aust. Jour. Sci. Res.*, vol. 4A, pp. 258-267; September, 1951.

Correspondence

ionospheric irregularities, during periods of heavy absorption when ionospheric echo-sounding techniques are unusable owing to the heavy attenuation of the reflected wave.

It is proposed to extend the present range of the experiments in order to investigate more fully the absorption and scintillation of extra-terrestrial radio waves. The use of these extra-terrestrial radio waves for ionospheric studies at high latitudes appears to have many advantages over normal echo-sounding techniques, e.g., the absence of the need to build and operate expensive radio transmitters; the use of higher frequencies enabling directional antennas to be built and hence permitting the investigation of a wide area of the ionosphere from a single station; the simplification of the results owing to the small effect of the earth's magnetic field on radio wave propagation at frequencies well above the gyro-frequency; the possibility of obtaining information about regions of the ionosphere above the level of maximum electron density; and the opportunity of studying ionospheric irregularities at times when frequencies below the critical frequency are heavily absorbed.

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Power Flow and Equivalent Circuits of Traveling-Wave Tubes*

C. C. Wang† to J. R. Pierce‡, September 30, 1953

In this equivalent circuit of traveling-wave tubes, using the notation (Fig. 1) in your book,¹ but, adding losses to the line, we have

$$\frac{\partial i}{\partial z} + \frac{\partial I}{\partial z} = -V(j\omega C + G) \quad (1)$$

$$\frac{\partial V}{\partial z} = -I(j\omega L + R). \quad (2)$$

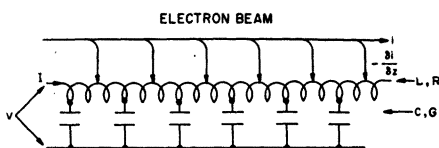


Fig. 1

we take the power products

$$(1)^*V + (1)V^* + (2)^*I + (2)I^*$$

then we have

$$\frac{1}{4} \left\{ \frac{\partial(VI^* + V^*I)}{\partial z} + \left(V \frac{\partial i^*}{\partial z} + V^* \frac{\partial i}{\partial z} \right) \right\} = -\frac{1}{2} (I^2 R + V^2 G).$$

Since the first term represents the variation

of power flow in the circuit, and the last term represents the power dissipation in the line, it is evident that the product,

$$-\frac{1}{4} \left[V \left(\frac{\partial i^*}{\partial z} + V^* \left(\frac{\partial i}{\partial z} \right) \right) \right]$$

must represent the power fed into the line by the beam.

On the other hand, the rate of change of kinetic energy of the beam with respect to the distance must be the power product of the electric field $\partial V/\partial z$ and the current i and this amounts to

$$\left[\left(\frac{\partial V^*}{\partial z} i + \frac{\partial V}{\partial z} i^* \right) \right] \frac{1}{4}.$$

Thus we face the problem of nonequality of these two energy expressions. The discrepancy just amounts to

$$\frac{1}{4} \frac{\partial(V^*i + Vi^*)}{\partial z}$$

which is the algebraic sum of the two. The question is whether we should consider the product $\frac{1}{4}(V^*i + Vi^*)$ as a sort of energy flow in order to keep the energy balance sheet.



Fig. 2



Fig. 3

This energy flow exists as long as V and i are coupled. If so, what happens to this energy when V and i are suddenly decoupled such as at the end of the tube (Fig. 2)? Or, if a current is suddenly introduced at the beginning of the circuit with a voltage V (Fig. 3)?

According to the equivalent circuit at these junctions there are large $\partial i/\partial z$ introduced at the circuit and large electric field variations $\partial V/\partial z$ at the beam, but presumably they are finite, integrable within the transition section. Therefore, at these junctions, you would expect some energy interchange between the beam and the circuit.

On the other hand, if $\frac{1}{4}(V^*i + Vi^*)$ cannot be considered as energy flow, then something must be done to restore the energy balance. This requires an extra condition to be imposed upon the traveling-wave tube equation.

I might add that if this is a discrepancy at all, it only amounts to an order of C times the normal circuit energy. Therefore, it will

be negligible anyway, if we assume C is small and so on. However, in the so-called large C cases, then this is just the order of magnitude we are dealing with in making our calculations.

J. R. Pierce to C. C. Wang, October 4, 1953

I think we should regard P_e .

$$P_e = \frac{1}{4}(V(I + i)^* + V^*(I + i))$$

as giving the total electromagnetic power flow, as you suggest, so that for a lossless circuit our equation is

$$\left(\frac{\partial P_e}{\partial z} - \left(i^* \frac{\partial V}{\partial z} + i \frac{\partial V^*}{\partial z} \right) \right) = 0.$$

Suppose we have, as shown in Fig. 1, a helix with a wire current I and a beam current i , ending in a tube at helix voltage into which the beam current i flows without being acted on by a longitudinal field.

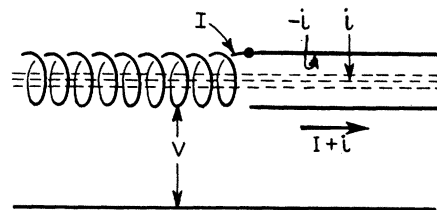


Fig. 1

Inside the tube there will be a beam current i and a wall current $-i$. On the outside of the tube there will be a current $I + i$. Hence, the circuit power delivered by the current flowing on the tube against the voltage V will be

$$\frac{1}{4}(V(I + i)^* + V^*(I + i))$$

just as we have assumed. The integral of

$$\frac{1}{4} \left(i^* \frac{\partial V}{\partial z} + i \frac{\partial V^*}{\partial z} \right)$$

from the beam source to the tube will be left in the beam as kinetic energy.

C. C. Wang to J. R. Pierce, November 13, 1953

Suppose we have a waveguide in which a free TM mode exists and it has a propagation constant γ_m , such that $\beta_m^2 = \gamma_m^2 - \gamma_0^2$ where $\gamma_0^2 = -\omega^2\mu\epsilon$, $\gamma_0 = \sqrt{\mu\epsilon}$ and if E_{zm} is the longitudinal component of the electric field, it will satisfy the equation

$$\nabla^2 E_{zm} + \beta_m^2 E_{zm} = 0$$

where ∇ represents the nabla operator in the transverse direction only. The transverse electric field will be proportional to ∇E_{zm} while the transverse magnetic field will be proportional to $\hat{k} \times \nabla E_{zm}$, where $\hat{k}$ is a unit vector in the z direction. Let us assume any electric field and magnetic field can be ex-

* Received by the IRE, August 17, 1954.
† Sperry Gyroscope Co., Great Neck, L. I., N. Y.
‡ Bell Telephone Labs., Inc., Murray Hill, N. J.
1 J. R. Pierce, "Traveling-Wave Tubes," D. Van Nostrand Co., New York, N. Y., p. 9; 1950.

Correspondence

panded in a series of modes which read like

$$\vec{E} = -A_m \frac{\gamma_m}{\beta_m^2} \nabla E_{zm} + \vec{k} B_m' E_{zm} + \dots \quad (1)$$

$$\vec{H} = -B_m \frac{\gamma_0}{\beta_m^2 Z_0} \vec{k} \times \nabla E_{zm} + \dots \quad (2)$$

where we use the contraction of the index m to indicate the summation of all the modes and $+\dots$ indicates that the series is not complete, e.g., there are TE modes present. However, for this discussion, we only have to pay attention to one TM mode at a time. $Z_0 = \sqrt{\mu/\epsilon}$.

A_m, B_m, B_m' are three arbitrary functions of z only while E_{zm} is understood to be functions of the transverse space co-ordinates only.

For simplicity, let us assume that we only have a longitudinal current in the traveling-wave tube so that the current density i_d can be expanded as a series of

$$i_d = \vec{k} J_m E_{zm} + \dots \quad (3)$$

J_m is again a function of z only.

If we try to substitute these into the Maxwell's curl equations, we must have

$$B_m' = B_m - \frac{Z_0}{\gamma_0} J_m \quad (4)$$

$$\frac{dB_m}{dz} + \gamma_m A_m = 0 \quad (5)$$

$$\frac{dA_m}{dz} + \gamma_m B_m = \frac{\beta_m^2}{\gamma_m} J_m \frac{Z_0}{\gamma_0} \quad (6)$$

If we further limit our interest to a filament of current at a particular location of the waveguide, so that the current i is i_d multiplied by a small area a and at this point E_z assumes a particular value of E_{zm0} , thus, by (3) we have $i_d = \vec{k} i/a$.

$$J_m = \frac{\iint \frac{i}{a} E_{zm} da}{\iint E_{zm} E_{zm} da} = \frac{i E_{zm0}}{\iint E_{zm}^2 da} \quad (7)$$

Define also an impedance Z such that

$$Z = \frac{\beta_m^2}{\gamma_m^3} \frac{Z_0}{\gamma_0} \frac{E_{zm0}^2}{\iint E_{zm} E_{zm} da} \quad (8)$$

Let us also renormalize A and B into voltage V and current I , such that

$$V = \frac{E_{zm0}}{\gamma_m} A_m$$

$$I = \frac{E_{zm0}}{\gamma_m Z} B_m \quad (9)$$

The reorganized differential equations then take the following forms:

¹ The following derivation is strictly true only for nonlossy waveguides so that γ_m and β_m are pure imaginary and Z is real. For the lossy case, other definitions of current, voltage, and impedance must be used and the equivalent circuit will be changed accordingly.

$$\frac{\partial V}{\partial z} + \gamma_m Z I = \gamma_m Z i \quad (10)$$

$$\frac{\partial I}{\partial z} + \frac{\gamma_m}{Z} V = 0 \quad (11)$$

Furthermore, the axial electric field at the beam

$$E_z \text{ at beam} = B_m' E_{zm0}$$

$$= \gamma_m Z I - \left[\frac{E_{zm0}^2 \frac{Z_0}{\gamma_0}}{\iint E_{zm} E_{zm} da} \right] i \quad (12)$$

The above three equations suggest the equivalent circuit for a traveling-wave tube, as in Fig. 1. According to this, the beam and the circuits are coupled together through a distributed mutual impedance $\gamma_m Z$ equal to the distributed impedance of the line itself. Also, the beam appears to have a self-

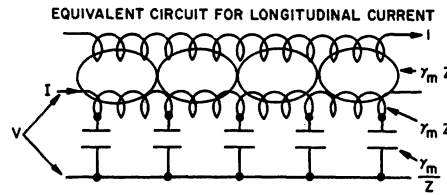


Fig. 1

impedance which is the bracket term in (12). The electric field induced by a current i into the line is $i \gamma_m Z$ while the electric field due to the line current I at the beam is $I \gamma_m Z$. The self-induced field at the beam due to its own current is reactive in nature since Z_0/γ_0 is essentially imaginary for nonlossy dielectrics. This self-induced field is in the nature of a space charge force. I believe since the space charge forces are nonsolenoidal, the space charge force represented by the second term of (12), which is an expansion in terms of a solenoidal mode alone, will not be complete. We may add an arbitrary nonsolenoidal and irrotational field without affecting the above equivalent circuit. I suppose in the end it can all be lumped into one single term proportional to the square of the plasma frequency as it was usually done. However, much remains to be done to clear the point.

As to the energy interchange, it is simply the time average of $\gamma_m Z i$ and I . The electromagnetic power flow will be represented by the time average of VI . This checks with the Poynting Vector product of (1) and (2) provided that we assume all the energy is contained in the TM mode. I suppose if a mixed mode is present instead of (8), the impedance Z can be redefined by using the actual value of the Poynting Vector including the TE component. The conservation of energy is seen to be preserved when we take proper products of (10), (11), and (12).

Now, if we make a further transformation

$$I = I' + \quad (13)$$

then (10) and (11) become

$$\frac{\partial V}{\partial z} + (\gamma_m Z) I' = 0 \quad (14)$$

$$\frac{\partial I'}{\partial z} + \frac{(\gamma_m) V}{Z} = - \frac{\partial i}{\partial z} \quad (15)$$

This immediately puts in a form equivalent to your original circuit. The fact that this transformation leaves the second order wave equation in V unchanged makes it appear that (14) and (15) can also serve as the basic equivalent circuit equations. However, this is so provided that we have to recognize the electromagnetic power as $V(I' + i) = VI$ instead of VI' , as we discussed before. I' can not then be considered as the complete circuit current.

In conclusion, the above derivation is basically the same as your original derivation of normal mode equation except that the coefficients are left in the equations and two basic transmission line (10) and (11) are directly obtained, instead of one single wave equation. These equations suggest that we should use another form of equivalent circuit so that we can truthfully say that the circuit voltage-current product represents all the electromagnetic power flow there is.

This derivation also shows that the circuit voltage V is simply a measure of the transverse electric field and the circuit current I is a measure of the transverse magnetic field. I have the difficulty of trying to relate the equivalent circuit current to any actual localized current flow along metallic conductors. It is easy to do in the case of a TEM mode of coaxial transmission, since this is undoubtedly the same as the current actually flowing along the conductors.

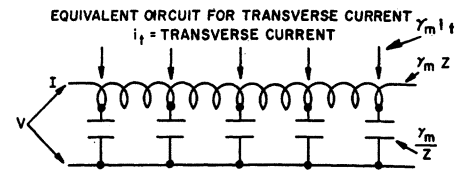


Fig. 2

However, in the case of a mixed mode like that of a helix, we can not fix a particular point where we can make a measure of this equivalent circuit current. Certainly it is not the current flowing along the helix wire. Therefore, to relate this current to actual beam current by using the divergence equation and the equation of continuity of currents (displacement current and conduction current), will need further clarification.²

Adding to the above derivation, when transverse currents are present, we may draw the equivalent circuit as in Fig. 2.

² Pierce, *ibid*.

P. K. Tien† to C. C. Wang, March 9, 1954

One thing which bothers me is as follows: We always have a conception that the coupling between the beam and the circuit is

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capacitive, and the beam generally acts as an inductive element to the axially directed electric field. In your equivalent circuit, however, the coupling turns out to be inductive.

A study of your (10), (11) and (12) indicates that in order for these equations to be consistent with your equivalent circuit, the mutual impedance is $\gamma_m z$ which is inductive and

$$\frac{\partial V}{\partial z} + \gamma_m z I + \gamma_m z (-i) = 0. \quad (10)$$

The use of $-i$ instead of i in the above equation puzzles me, although as the equations stand, it seems straightforward that the $-i$

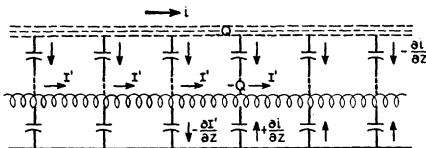


Fig. 1

of the electron beam acted with the mutual impedance $\gamma_m z$, enters the transmission line equation (10) and so in your equivalent circuit. It may however confuse the beginner why $-i$ should be used instead of i .

To explain this, we say that the coupling is actually capacitive. Let's draw a physical picture. Assuming that the coupling is capacitive, any positive charge in the beam will induce a negative charge on the circuit. When the beam current varies, the charge on the circuit varies accordingly and this is accomplished by an additional current flowing on the shunt element of the circuit in a direction from ground to the circuit, as illustrated in Fig. 1, which of course is the Pierce original equivalent circuit. As mentioned by Pierce, without the electron stream, the electromagnetic power is VI^* . Because of the electron stream and the capacitive coupling, the total current flow in the shunt element is $\partial(I' + i)/\partial z$ and so the power is $V(I' + i)^*$.

Your equivalent circuit traces out very clearly the relation between the power-flows, except that the use of $-i$ and the inductive coupling may mislead with regard to the actual physical concept.

We should note that in your first letter I is the circuit current as defined in Pierce's book while in your second letter I' is the same current. Our notation follows that in your letters.

A Rapid Method for Matching High Frequency Transmission Systems*

Recent developments in the manufacture of dielectric materials make the following a

simple and effective method for rapid matching in waveguide and other high frequency transmission systems. A slug of dielectric material having the same cross section as the guiding system, and a quarter wavelength long (in the dielectric) is inserted into the guide. The dielectric constant is chosen so that the slug produces a vswr equal to the mismatch which is to be eliminated. The slug is then moved along the guide until a match is achieved.

The vswr produced by such a dielectric slug may be calculated either by requiring continuity of tangential E and H across the two surfaces of the dielectric, or by a consideration of the wave impedances normal to the surfaces. The result of either calculation is that the dielectric constant required to produce a given standing wave ratio "s" is given by

$$\epsilon = s \left[1 - \left(\frac{\lambda}{\lambda_c} \right)^2 \right] + \left(\frac{\lambda}{\lambda_c} \right)^2$$

where λ_c is the free space cut-off wavelength of the guiding system, λ is the free space operating wavelength, and ϵ is the relative dielectric constant of the slug. It is to be noted that this relation is fairly general; it holds for all TE waves in any uniform cylindrical guiding system, i.e., systems which can be generated by translating a cross-section parallel to a fixed straight line.

It is interesting that the standing-wave ratio and the dielectric constant are related linearly. This simple relationship does not hold between the reflection coefficient and the dielectric constant, however, hence the reflection coefficient is not considered further.

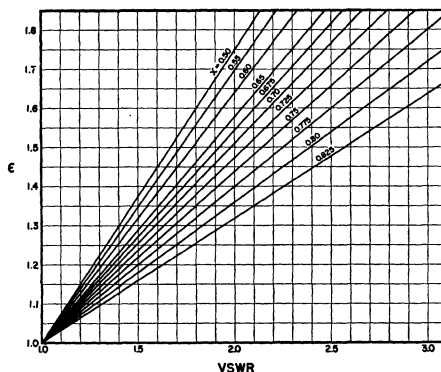


Fig. 1

In Fig. 1, the dielectric constant is plotted as a function of standing-wave ratio for various values of $x = \lambda/\lambda_c = f_c/f$. To use the curves, first calculate x from the operating and cut-off wavelengths or frequencies. This singles out one of the straight lines on the graph. Using the vswr it is desired to match out as the abscissa, read off the required value of ϵ on the ordinate scale. The slug is then made of material having this

dielectric constant and should have an axial length

$$l = \frac{\lambda}{4\sqrt{\epsilon - \left(\frac{\lambda}{\lambda_c}\right)^2}}.$$

As previously indicated, the match is then achieved by sliding the slug along the guide. If desired, the slug can be fixed in its final position by a transverse pin perpendicular to the E -vector or by other means.

For TEM systems, $\lambda_c = \infty$, and a simple result is obtained:

$$\epsilon = s$$

$$l = \frac{\lambda}{4\sqrt{\epsilon}}.$$

Additional curves may easily be added to the figure in the general case, or the range may be extended, by simply calculating a single point and joining it to the point 1, 1. Actually it is almost as simple to make the calculation directly from the formula in each specific case.

Present dielectric materials are available in steps of 0.05 from $\epsilon=1$ to $\epsilon=2.6$. Power handling capability is moderate with present materials due to accumulation of heat in the cellular structure of the dielectric. Contemplated improvements in these materials should reduce the loss tangent to a point where internal heating is almost negligible. For moderate standing-wave ratios this method should then be satisfactory even for very high power levels. For very large mismatches, and hence high dielectric constants, the method will not be suitable for high power levels even with perfect dielectrics, since breakdown between the dielectric and guide wall will become a factor. However, this might be overcome by pressurizing or by using two methods of matching in conjunction.

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Adventures in Calculus*

We have recently encountered two new mathematical results of interest:

$$\int \frac{dc}{cabin} = \log cabin + C = \text{house boat}$$

$$d(Hi Ho) = Hi dHo + Ho dHi.$$

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* Received by the IRE, August 12, 1954.

* Received by the IRE, July 28, 1954.

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Ratio Squarer*

During a recent symposium on military communications, a number of papers¹⁻⁴ discussed in part a new form of diversity reception identified as a "Ratio Squarer." Kindly reference and acknowledgment was made to the undersigned as the originator of this system, with a request for publication of the analysis which pointed the way to this development.

The following is accordingly a simplified analysis, which may be of interest to members of the Institute.

It was long felt that the sharper the diversity selection the better the operation of the system. Therefore, many workers have attempted to improve the sharpness of selection of diversity systems, and it is now possible to practically eliminate the weaker diversity signals and accept only the strongest with differences in diversity levels of less than a decibel. It is obvious that this mode of operation produces a signal-to-noise ratio equal to the best diversity signal at all times.

It appeared evident, however, to the writer that some thought should be given as to whether the goal of sharp diversity selection is a desirable one. The following analysis indicates that it is not desirable to completely discard the weaker diversity signals, because use may be made of them to improve the over-all signal-to-noise ratio. The maximum improvement, possible from this new expedient, is 3 db for two-receiver diversity, 4.8 db for three-receiver diversity and 6 db for four-receiver diversity. These figures are optimum values and occur only when the signals being received are equal in amplitude. When they differ from equality, the improvement decreases but at all times the combined signal-to-noise ratio is greater than the best of the individual diversity signal-to-noise ratio.

In order to determine the optimum diversity combining law, the following question must be answered. For a given ratio of diversity signal levels, how much of the weaker signal with its accompanying noise should be added to the stronger signal and its noise if an optimum signal-to-noise ratio is to be obtained? It is assumed that the signals are combined so that they add in phase, whereas the noise adds in a root-mean-square fashion. This may be readily accomplished by connecting the dc outputs of the signal demodulators in series.

Assumptions:

1. Signal from the diversity channels add linearly.

2. The noises add root-mean-square.
3. Two-receiver diversity is utilized.
4. The noise in one diversity receiver equals the noises from the other receiver.
5. The noise is random (white) in character.

D is the factor by which the weaker channels signal plus noise is divided in order to optimize the combined output signal-to-noise ratio. It is assumed that the gain of the stronger channel is unity and therefore D is equal to the gain of the stronger channel over the gain of the weaker channel.

To determine the value of D for maximum combined signal-to-noise ratio, it is necessary to differentiate the combined signal-to-noise ratio with respect to D . This differential is then set equal to zero in order to determine the optimum value of D .

$$S/N = \frac{S_1 + S_2/D}{\sqrt{N_1^2 + N_2^2/D^2}} \quad (1)$$

But it is assumed that $N_2 = N_1$

$$S/N = \frac{S_1 + S_2/D}{\sqrt{N_1^2 + N_1^2/D^2}} \quad (2)$$

$$= \frac{DS_1 + S_2}{N_1\sqrt{D^2 + 1}} \quad (3)$$

$$\frac{dS/N}{dD} = \frac{N_1(D^2 + 1)^{1/2}S_1 - (DS_1 + S_2) \frac{N_1}{2} (D^2 + 1)^{-1/2}D}{N_1^2(D^2 + 1)} = 0 \quad (4)$$

$$(D^2 + 1)S_1 = (DS_1 + S_2)D \quad (5)$$

$$D = \frac{S_1}{S_2} \quad (6)$$

Where S_1 is the signal from one receiver and S_2 the signal from the second channel, N_1 is the noise accompanying S_1 and N_2 accompanying S_2 .

It is therefore apparent that, in order to optimize the combined output signal-to-noise ratio in a system wherein the signal is made to add linearly and the noise root-mean-square, the ratio of the signal levels should be squared and then summated. In other words, if the signals were 4 db different in level, the ratio should be increased to 8 db before combination. Heretofore, it has been assumed that the weaker signal should be completely eliminated, but we see that this does not optimize the signal-to-noise ratio.

Fig. 1 shows a possible method for obtaining the ratio squarer selection curve (used by the U. S. Army Signal Corps in the AN/FGC-29 equipment). It is shown for a two-receiver diversity system, however, the basic arrangement may be readily adapted for three- or four-receiver diversity. It is seen that the signal and noise from receiver 1 are fed to a variable gain amplifier 1 which in turn feeds diode rectifier $D1$. The signal and noise from receiver 2 are fed to variable gain amplifier 2, which in turn feeds diode $D2$. Diode $D1$ and $D2$ are connected to a common load and the diode fed by the stronger signal conducts and biases the other diode off. Therefore, the output

voltage across resistor R_{CL} is determined solely by the level of the stronger channel. This voltage is then fed through a lowpass filter and it is used to control the gain of the variable gain amplifiers 1 and 2. Thus, the output from the variable gain amplifier fed by the stronger signal is maintained constant. The gains of variable gain amplifier 1 and 2 are equal and there is no selection in this part of the circuit. It will be recognized that this is identical with the conventional common AVC diversity system.

The outputs of the individual variable gain amplifiers are fed to separate circuits which vary their gain linearly with signal

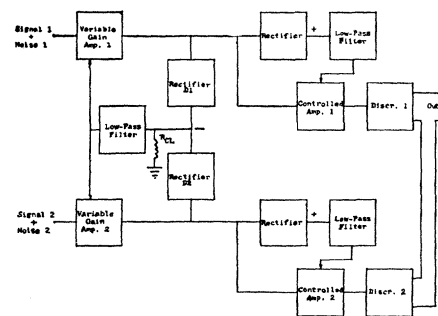


Fig. 1

level. By this process, the level difference between the weaker and stronger signals is squared. The manner in which this squaring is done is readily seen if one considers the circuit to be equivalent to a linear volume expander. A portion of the signal is rectified and the rectifier connected so that the bias on the controlled amplifier decreases with an increase in signal level. With this circuit, the gain may be made a direct function of the input level and therefore the ratio of the output of the two channels is proportional to the square of the input signal levels. The output of these amplifiers is then fed to demodulators (discriminators in the case of FSK reception) and then the outputs of the discriminators are connected in series giving the desired linear signal addition and root-mean-square noise addition effect.

Many tests were performed evaluating and comparing this system and the results were extremely gratifying. When compared with sharp selection-type diversity system and the common limiter diversity system, as well as other diversity systems, an appreciable reduction in error count was achieved and, in fact, a reduction in error count of 26 per cent was demonstrated over the next best system (at an error rate of approximately 2 errors per thousand characters).

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* Received by the IRE, July 13, 1954. This work was done as part of Contract No. DA36-039 sc-15359, administered by the Long-Range Section of Coles Signal Lab.

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